Strut-and-Tie Models for Deep Beam Design

What happened to the simplicity of the truss analogy?

BY GUSTAVO J. PARRA-MONTESINOS

Truss models have long been used as tools for the analysis and design of reinforced concrete members. Late in the nineteenth century, Hennebique had already envisioned the use of truss models for the calculation of forces in stirrups.[1] Using a truss analogy, a beam containing vertical stirrups is modeled by chords representing the flexural compression and tension regions of the beam, vertical web members representing the stirrups, and inclined web members representing the web concrete (Fig. 1).

Lower shear strengths predicted using the truss analogy compared to test results led researchers to attribute an additional shear resistance to the concrete,[2,3] generally as a function of the concrete compressive strength f_c'. This led to the general format adopted by ACI 318-08[4] for determining the nominal shear strength V_n of a reinforced concrete beam with web reinforcement

$$V_n = V_c + V_s \quad (1)$$

where V_c has been traditionally referred to as the concrete shear strength, intended to lump contributions from the compression zone, aggregate interlock, and dowel action, and V_s is the shear strength contribution from the web reinforcement. In reality, however, these two resistances are interrelated and not independent, as implied by Eq. (1). In ACI 318-08, the calculation of V_s is based on an assumed projection of the diagonal crack on the longitudinal reinforcement plane equal to the beam effective depth d. The same result is obtained using the 45-degree truss analogy envisioned by Mörsch.[5]

In the past 20 years, the use of the truss analogy to analyze and design regions of members, particularly those in which the assumption of plane sections remaining plane after loading is clearly inadequate, has been the subject of considerable research.[6-10] In these works, limit analysis is used to evaluate the strength of the compression strut and tension tie members of the truss, as well as the nodes connecting them. These truss models are thus typically referred to as strut-and-tie models.

Although truss or strut-and-tie models have long served as the basis of shear design methods for beams with web reinforcement, design standards such as the AASHTO LRFD Bridge Design Specifications,[11] CSA Standard A23.3-04,[12] and ACI 318-08 have incorporated formal provisions for the use of strut-and-tie models.

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An extended design example is included with the electronic version of this article at www.concreteinternational.com.

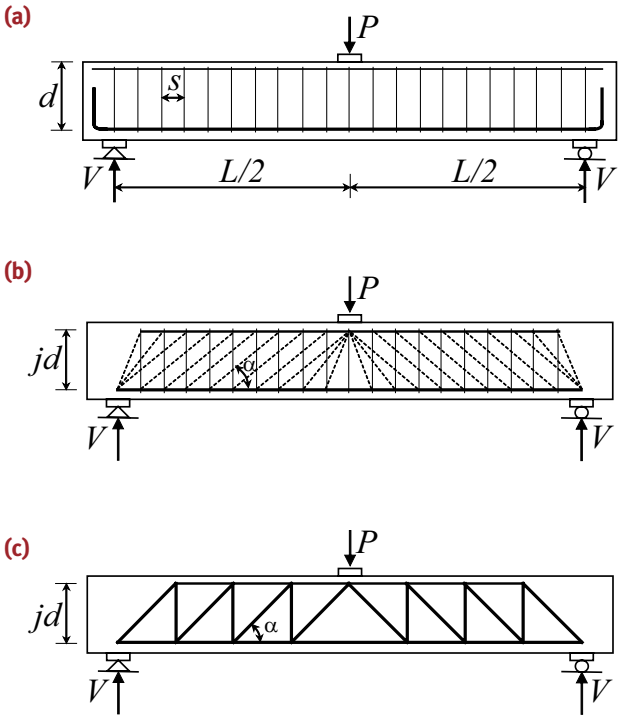


Fig. 1: Illustration of the truss analogy applied to a reinforced concrete beam: (a) beam reinforcement; (b) a refined truss model with one vertical tie member per stirrup; and (c) a simplified truss model

Because these provisions typically include rules for estimating the size and strength of all the components (struts, ties, and nodes) in truss models, design using the strut-and-tie approach is often tedious and time-consuming. This effectively compromises one of the pillars of the truss analogy: simplicity.

In my opinion, simplification of the strut-and-tie method is needed before it can be widely implemented as a tool for design. Toward this end, I will attempt to show that, in many cases, design of deep beams using the strut-and-tie method can be substantially simplified using simple rules that have passed the test of time.

TRUSS ANALOGY IN THE CODE

Figure 1(a) shows a simply supported beam with a concentrated force at midspan and vertical stirrups spaced at a distance s. Using the 45-degree truss analogy proposed by Mörsch[5] ($\alpha = 45$ degrees in Fig. 1(b) and 1(c)), the force in each stirrup F_v can be calculated as

$$F_v = \frac{s}{jd}V \tag{2}$$

where V is the applied shear and jd is the depth of the truss. The maximum force in each stirrup is governed by either the strength of the stirrup itself (typically assumed to be reached at yielding of the steel), or by the strength of the diagonal members representing the concrete web. In terms of an average shear stress $v = V/(bjd)$, where b is the beam web width, these two limit states can be expressed as

$$v \le \frac{F_{vy}}{sb} \tag{3a}$$

$$v \le v_{max} \tag{3b}$$

F_{vy} is the strength of the stirrup legs at yield and v_{max} is the shear stress corresponding to failure of the diagonal concrete (web) members.

In ACI 318-08, the shear force V in Eq. (2) corresponding to the maximum force in the stirrups is equivalent to V_s, assuming $jd \approx d$. It's interesting to note that this strut-and-tie model on which V_s is based does not include checks for the capacity of struts and nodes, as required in Appendix A of ACI 318-08. Instead, failure of the web concrete is prevented by limiting either the average shear stress resisted by the web reinforcement, v_s, or the total shear stress to a value that is a function of the concrete compressive strength. For slender beams, ACI requires $v_s \le 0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi), while for deep beams the total shear stress is limited to $0.83\sqrt{f_c'}$ in MPa ($10\sqrt{f_c'}$ in psi). The AASHTO LRFD Bridge Design Specifications[11] and Canadian Standards[12] follow a similar philosophy, but the total shear stress is limited to $0.25f_c'$, which is a substantially higher shear stress limit compared with that in ACI 318-08.

The ACI shear stress limits are largely based on a report by Joint ACI-ASCE Committee 326[13] indicating that $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi) represented a safe limit for the total shear stress in rectangular beams to prevent shear-compression failures (that is, crushing of the concrete at the tip of the critical diagonal crack) prior to yielding of the web reinforcement. A limit of $0.83\sqrt{f_c'}$ in MPa ($10\sqrt{f_c'}$ in psi) was found to be adequate for T-beams with vertical stirrups. These limits were primarily based on test results from deep beams that, in ACI 318-08, are required to be designed using the strut-and-tie method. This limit was also found to be conservative for preventing web-crushing failures.[14]

PROPOSED SIMPLIFICATION

Determining the geometry and nominal strength of struts and nodes in strut-and-tie models is a time-consuming task that is often not justified, particularly when designing normal-strength concrete beams with $f_c' \le 41$ MPa (6000 psi) containing at least minimum web reinforcement. Considering that the use of a shear stress limit to prevent shear-compression (prior to stirrup yielding) and web-crushing failures in either slender or deep beams has been accepted practice for decades, it's odd that designers are required to check the strength of idealized struts and nodes in beams subjected to shear stresses lower than $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi). In my opinion, the check for stresses in struts and nodes in the beam design should

only be required when shear stresses are above these proven, safe limits; in other words, when designers want to use a higher strength for design.

By eliminating the calculation of stresses in struts and nodal zones, the design of deep beams will become substantially simpler while we preserve the principal function of the truss analogy: to allow determination of tension forces in reinforcement.

SHEAR STRESS LIMIT FOR DEEP BEAMS

Besides the evidence provided in the Joint ACI-ASCE Committee 326 report[13] with regard to maximum shear stress limits to prevent shear-compression failures, a database of deep beam tests[15] is used to support a shear stress limit of $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi) above which strength checks for struts and ties are not necessary. Only normal- strength concrete beams with $f_c' \leq 41$ MPa (6000 psi) are considered because of experimental evidence that indicates higher web reinforcement ratios are required for high-strength concrete deep beams.[16] Except for a few, the beams reported in the database contained web reinforcement satisfying the minimum percentage specified for normal-strength concrete in Appendix A of the 2008 ACI Code as follows

$$\sum \frac{A_{si}}{bs_i} \sin \gamma_i \geq 0.003 \qquad (4)$$

where A_{si} is the area of reinforcement at a spacing s_i for reinforcement layer i oriented at an angle γ_i with the strut axis.

Figure 2 shows a plot of normalized maximum shear stress versus shear span-to-effective depth ratio a/d for the beams in the database with $f_c' \leq 41$ MPa (6000 psi). As expected, the general trend is a decrease in shear strength with an increase in a/d. For a/d below 1.75, however, experimental shear strengths were greater than or approximately equal to the upper shear stress limit of $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi). For most practical cases, an a/d of 1.75 would correspond to an angle between the diagonal compression strut and the main tie (α in Fig. 3(a)) near the minimum of 25 degrees specified in Appendix A. It's worth mentioning that the test results used for validation of the proposed simplification should not only be applicable to deep beams, but also to members (or regions of members) where shear is primarily resisted by the action of a single diagonal strut, such as corbels.

ADDITIONAL CONSIDERATIONS

Although the proposed simplification eliminates the need for checking stresses in struts and nodal zones, the check for bearing stresses induced by the application of concentrated forces on deep beams should still be performed. For this purpose, a permissible concrete bearing stress equal to $0.85f_c'$ can be assumed, based on

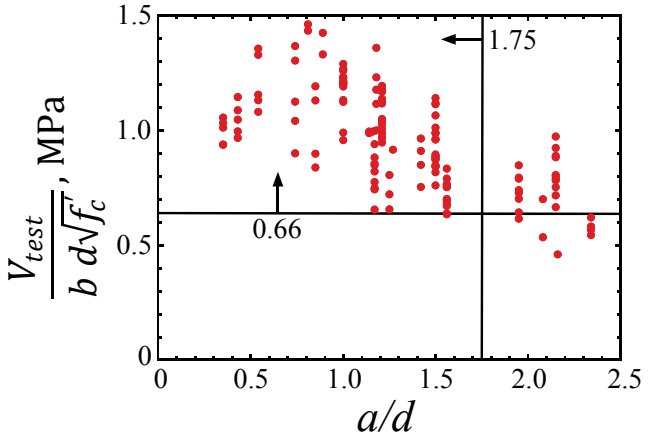


Fig. 2: Normalized maximum shear stress versus a/d for normal-strength concrete beam data taken from Reference 15. V_{test} is the experimental failure load (1 MPa = 145 psi)

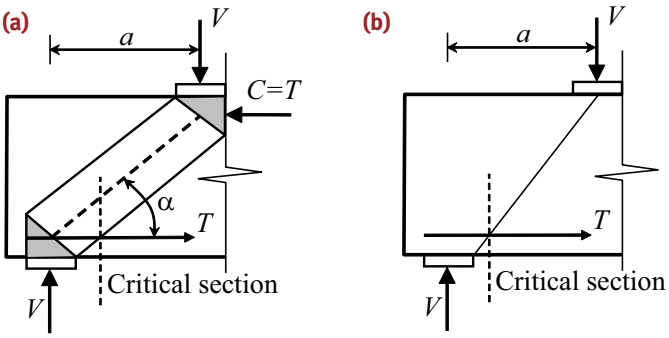


Fig. 3: Critical section for anchorage of tie reinforcement: (a) as defined in ACI 318-08; and (b) as proposed in the simplified approach presented in this article

Section 10.14.1 of ACI 318-08. Also, because the geometry of nodal zones and struts would not be determined, anchorage of the main tie reinforcement cannot be checked using an extended nodal zone, as allowed in Appendix A and illustrated in Fig. 3(a). Reinforcement anchorage, however, may be conservatively taken either from the intersection between the centroid of the tie reinforcement and a line joining the inner face of the support and the load point, as illustrated in Fig. 3(b), or simply from the face of the support.

DESIGN EXAMPLE

To illustrate the proposed simplified method, the beam shown in Fig. 4(a) will be designed for a concentrated force P_u = 780 kN (175 kip). Strength reduction factors ϕ will be neglected. The calculations are based on a concrete compressive strength of 28 MPa (4000 psi) and a steel yield strength of 420 MPa (60 ksi). For comparison, the design using Appendix A of ACI 318-08 is included with the electronic version of this article at **www.concreteinternational.com**.

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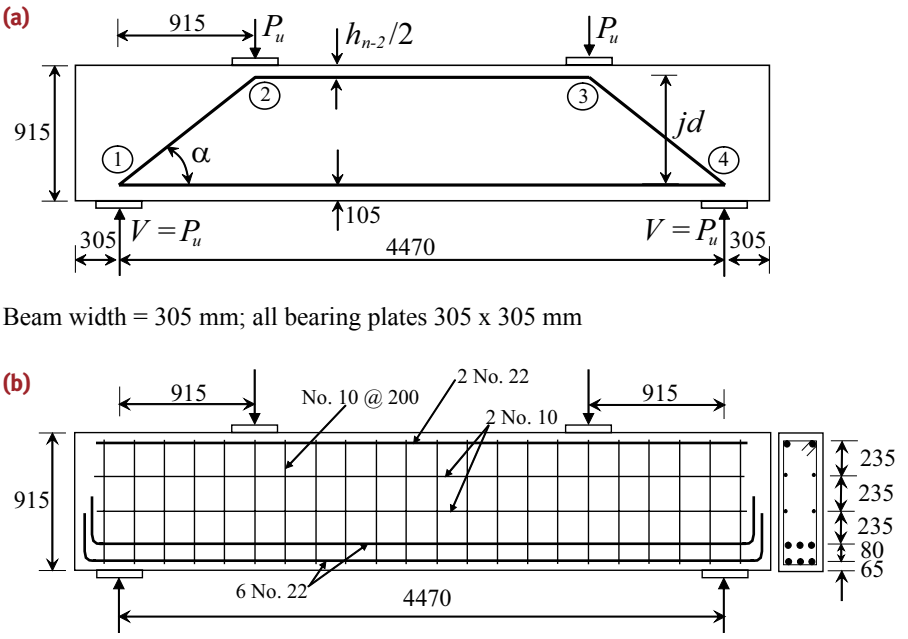


Fig. 4: Design example beam: (a) beam geometry and strut-and-tie model; and (b) final beam design (all dimensions in millimeters, 1 in. = 25.4 mm)

The first step is to verify that a/d is less than or equal to 1.75. From Fig. 4(a), the shear span a is 915 mm (36 in.) and the effective depth d = 915 mm – 105 mm = 810 mm (d = 36 in. – 4.1 in. = 31.9 in.). Therefore,

$$a/d = 915 \text{ mm}/810 \text{ mm} = 1.13; a/d < 1.75 \ \therefore \text{OK} \quad \text{(SI units)}$$

$$a/d = 36 \text{ in.}/31.9 \text{ in.} = 1.13; a/d < 1.75 \ \therefore \text{OK} \quad \text{(in.-lb. units)}$$

Next, check that the bearing stress f_b is less than $0.85f_c'$ at the load and support locations using a bearing area A_b that is 305 x 305 mm (12 x 12 in.).

$$f_b = \frac{P_u}{A_b} = \frac{780 \text{ kN}}{(305 \text{ mm})(305 \text{ mm})} = 8.4 \text{ MPa}; f_b < 0.85f_c' \ \therefore \text{OK} \quad \text{(SI units)}$$

$$f_b = \frac{P_u}{A_b} = \frac{175 \text{ kip}}{(12 \text{ in.})(12 \text{ in.})} = 1220 \text{ psi}; f_b < 0.85f_c' \ \therefore \text{OK} \quad \text{(in.-lb units)}$$

Also check that the applied average shear stress v does not exceed v_{max} = $0.66\sqrt{f_c'}$ =3.5 MPa (v_{max} = $8\sqrt{f_c'}$ = 505 psi).

$$v = \frac{V}{bd} = \frac{780 \text{ kN}}{(305 \text{ mm})(810 \text{ mm})} = 3.2 \text{ MPa}; v \le v_{max} \ \therefore \text{OK} \quad \text{(SI units)}$$

$$v = \frac{V}{bd} = \frac{175 \text{ kip}}{(12 \text{ in.})(31.9 \text{ in.})} = 460 \text{ psi}; v \le v_{max} \ \therefore \text{OK} \quad \text{(in.-lb units)}$$

To determine the forces in the struts and ties, assume a depth for the truss model equal to 0.9d, where d = 810 mm (31.9 in.). Thus, jd = 0.9 × 810 mm = 729 mm (jd = 0.9 × 31.9 in. = 28.7 in.) and α = arctan (729 mm/ 915 mm) = arctan (28.7 in./ 36 in.) = 38.6 degrees. Using this value of α, the forces in the truss members are calculated as follows:

$$F_{tie1-4} = F_{strut2-3} = 780 \text{ kN} \cot 38.6° = 979 \text{ kN} \quad \text{(SI units)}$$

$$F_{tie1-4} = F_{strut2-3} = 175 \text{ kip} \cot 38.6° = 220 \text{ kip} \quad \text{(in.-lb units)}$$

$$F_{strut1-2} = \frac{780\text{ kN}}{\sin 38.6^\circ} = 1252\text{ kN} \quad \text{(SI units)}$$

$$F_{strut1-2} = \frac{175\text{ kip}}{\sin 38.6^\circ} = 280\text{ kip} \quad \text{(in.-lb units)}$$

Because Node 2 connects struts only, $\beta_n = 1$ per Section A.5.2.1, and the depth of the depth of Nodes 2 and 3, labeled h_{n-2} in Fig. 4(a), can be determined as follows:

$$h_{n-2} = \frac{F_{strut2-3}}{0.85\beta_n f'_c b} = \frac{979\text{ kN}}{0.85(1)(28\text{ MPa})(305\text{ mm})} = 135\text{ mm} \quad \text{(SI units)}$$

$$h_{n-2} = \frac{F_{strut2-3}}{0.85\beta_n f'_c b} = \frac{220\text{ kip}}{0.85(1)(4\text{ ksi})(12\text{ in.})} = 5.4\text{ in.} \quad \text{(in.-lb units)}$$

Knowing h_{n-2}, the assumed depth of the truss can be checked by verifying that α is close to the value of 38.6 degrees calculated earlier:

$$\alpha = \tan^{-1}\frac{810\text{ mm} - \frac{135\text{ mm}}{2}}{915\text{ mm}} = 39.1\text{ degrees} \quad \text{(SI units)}$$

$$\alpha = \tan^{-1}\frac{31.9\text{ in.} - \frac{5.4\text{ in.}}{2}}{36\text{ in.}} = 39.1\text{ degrees} \quad \text{(in.-lb units)}$$

The two values for α are close enough that further iteration would yield minimal change in the calculated forces. Therefore, $jd = 729$ mm ($jd = 28.7$ in.) and $\alpha = 38.6$ degrees will be used for the rest of the example.

The next step is to calculate the required area of bottom tie reinforcement $A_{s(tie1\text{-}4)}$

$$A_{s(tie1-4)} = \frac{F_{tie1-4}}{f_y} = \frac{979\text{ kN}}{420\text{ MPa}} = 2330\text{ mm}^2 \quad \text{(SI units)}$$

$$A_{s(tie1-4)} = \frac{F_{tie1-4}}{f_y} = \frac{220\text{ kip}}{60\text{ ksi}} = 3.67\text{ in.}^2 \quad \text{(in.-lb units)}$$

Select six No. 22 (No. 7) bars ($A_s = 2322$ mm^2 [$A_s = 3.6$ in.^{2}] $\approx A_{s(tie1\text{-}4)}$). Check the anchorage length for these bars from the intersection between the centroid of the tie reinforcement and a line joining the inner face of the support plate and the location of the concentrated force applied on the beam top, as illustrated in Fig. 3(b). Assuming a 50 mm (2 in.) minimum cover at the end of the tie reinforcement, an available anchorage length of about 495 mm (19.5 in.) is obtained. The development length l_{dh} for a No. 22 (No. 7) hooked bar is

$$l_{dh} = \frac{0.24 f_y}{\sqrt{f'_c}} d_b = \frac{0.24(420\text{ MPa})}{\sqrt{28\text{ MPa}}}(22\text{ mm}) = 420\text{ mm} \quad \text{(SI units)}$$

$$l_{dh} = \frac{0.02 f_y}{\sqrt{f'_c}} d_b = \frac{0.02(60{,}000\text{ psi})}{\sqrt{4000\text{ psi}}}(0.875\text{ in.}) = 17\text{ in.} \quad \text{(in.-lb units)}$$

Finally, determine the minimum web reinforcement to satisfy Section A.3.3.1 of ACI 318-08. Try two-legged No. 10 (No. 3) stirrups at 200 mm (8 in.) spacing and two layers of two No. 10 (No. 3) horizontal bars at 235 mm (9.25 in.) spacing as shown in Fig. 4(b).

$$\sum \frac{A_{si}}{bs_i}\sin\gamma_i = \frac{142\text{ mm}^2}{(305\text{ mm})(200\text{ mm})}\sin(90^\circ - 38.6^\circ) + \frac{142\text{ mm}^2}{(305\text{ mm})(235\text{ mm})}\sin 38.6^\circ = 0.0031 \quad \text{(SI units)}$$

$$\sum \frac{A_{si}}{bs_i}\sin\gamma_i = \frac{0.22\text{ in.}^2}{(12\text{ in.})(8\text{ in.})}\sin(90^\circ - 38.6^\circ) + \frac{0.22\text{ in.}^2}{(12\text{ in.})(9.25\text{ in.})}\sin 38.6^\circ = 0.0030 \quad \text{(in.-lb units)}$$

$$\sum \frac{A_{si}}{bs_i}\sin\gamma_i \geq 0.003 \therefore \text{OK}$$



Although not required, add two No. 22 (No. 7) longitudinal bars as compression reinforcement for ductility. The final beam design is shown in Fig. 4(b).

The same beam design is obtained following either Appendix A of ACI 318-08 or the proposed simplified approach. Eliminating the need to check struts and nodal zones in the strut-and-tie method, however, greatly simplifies its application, even for this simple design example. A greater contrast between the two approaches would be seen in strut-and-tie models with nodes receiving three or more members (struts and/or ties), such as the one in the design example in Reference 17.

CONCLUSIONS

The proposed simplification to the ACI strut-and-tie model provisions for deep beam design is based on the truss analogy and previously published experimental data. This simplification consists of eliminating the check for stresses in struts and nodal zones if the following four conditions are satisfied: 1) normal-strength concrete with $f_c' \leq 41$ MPa (6000 psi) is used; 2) $a/d \leq 1.75$; 3) at least minimum web reinforcement as specified in Appendix A of ACI 318-08 is provided; and 4) average shear stress $v = V/bd$ is less than or equal to $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi). This shear stress limit has been reported by Joint ACI-ASCE Committees 326[13] and 426[14] to be safe to prevent shear-compression failures prior to yielding of web reinforcement, as well as web crushing failures.

Results contained in a recently published database[15] also support the use of this shear stress limit. Further, $0.66\sqrt{f_c'}$ in MPa ($8\sqrt{f_c'}$ in psi) is the current ACI Code stress limit for the shear contribution from web reinforcement in beams, contribution that is based on a strut-and-tie model. Thus, the proposed simplification should not be seen as a new idea, but as the use of an existing tool that has passed the test of time when applied to structural design using the truss analogy.

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DESIGN FOLLOWING APPENDIX A OF ACI 318-08

For comparison with the proposed simplified method illustrated in the article, the beam shown in Fig. 4(a) will be designed for a concentrated force P_u = 780 kN (175 kip) using the strut-and-tie method of ACI 318-08. Strength reduction factors ϕ will be neglected. The calculations are based on a concrete compressive strength of 28 MPa (4000 psi) and a steel yield strength of 420 MPa (60 ksi).

To determine the forces in the struts and ties, assume a depth for the truss model equal to 0.9d, where d = 810 mm (31.9 in.). Thus, jd = 0.9×810 mm = 729 mm (jd = 0.9×31.9 in. = 28.7 in.) and α = arctan (729 mm/ 915 mm) = arctan (28.7 in./ 36 in.) = 38.6 degrees. Using this value of α, the forces in the truss members are calculated as follows:

$F_{tie1-4} = F_{strut2-3} = 780 \text{ kN} \cot 38.6° = 979 \text{ kN}$ (SI units)

$F_{tie1-4} = F_{strut2-3} = 175 \text{ kip} \cot 38.6° = 220 \text{ kip}$ (in.-lb units)

$F_{Strut1-2} = \frac{780 \text{ kN}}{\sin 38.6°} = 1252 \text{ kN}$ (SI units)

$F_{Strut1-2} = \frac{175 \text{ kip}}{\sin 38.6°} = 280 \text{ kip}$ (in.-lb units)

Because Node 2 connects struts only, β_n = 1 per Section A.5.2.1, and the depth of the strut between nodes 2 and 3, labeled $h_{n\text{-}2}$ in Fig. 4(a), can be determined as follows:

$h_{n-2} = \frac{F_{strut2-3}}{0.85\beta_n f'_c b} = \frac{979 \text{ kN}}{0.85(1)(28 \text{ MPa})(305 \text{ mm})} = 135 \text{ mm}$ (SI units)

$h_{n-2} = \frac{F_{strut2-3}}{0.85\beta_n f'_c b} = \frac{220 \text{ kip}}{0.85(1)(4 \text{ ksi})(12 \text{ in.})} = 5.4 \text{ in.}$ (in.-lb units)

Knowing $h_{n\text{-}2}$, the assumed depth of the truss can be checked by verifying that α is close to the value of 38.6 degrees calculated earlier:

$\alpha = \tan^{-1} \frac{810 \text{ mm} - \frac{135 \text{ mm}}{2}}{915 \text{ mm}} = 39.1 \text{ degrees}$ (SI units)

$\alpha = \tan^{-1} \frac{31.9 \text{ in.} - \frac{5.4 \text{ in.}}{2}}{36 \text{ in.}} = 39.1 \text{ degrees}$ (in.-lb units)

The two values for α are close enough that further iteration would yield minimal change in the calculated forces. Therefore, jd = 729 mm (jd = 28.7 in.) and α = 38.6 degrees will be used for the rest of the example.

Next, the bearing stress $f_{bearing}$ are checked at Node 2:

$f_{bearing-2} = \frac{780 \text{ kN}}{(305 \text{ mm})(305 \text{ mm})} = 8.4 \text{ MPa}$ (SI units)

$f_{bearing-2} = \frac{175 \text{ kip}}{(12 \text{ in.})(12 \text{ in.})} = 1.2 \text{ ksi}$ (in.-lb units)

The maximum value of $f_{bearing\text{-}2}$ is limited to $0.85f'_c$ = 0.85×28 MPa = 23.8 MPa ($0.85f'_c$ = 0.85×4 ksi = 3.4 ksi) in section 10.14.1 of ACI 318-08. Because $h_{n\text{-}2}$ was determined based on the maximum allowable stress, there is no need to check the stress on the vertical face of Node 2.

Next, the stress on inclined face of Node 2 is checked. The width of the inclined face of Node 2 is the same as the width of Strut 1-2 at Node 2, $w_{strut\text{-}2}$ as in Fig. 5(a).

$w_{strut-2} = h_{n-2} \cos\alpha + w_{bearing} \sin\alpha$

$= (135 \text{ mm}) \cos 38.6° + (305 \text{ mm}) \sin 38.6° = 296 \text{ mm}$ (SI units)

$= (5.4 \text{ in.}) \cos 38.6° + (12 \text{ in.}) \sin 38.6° = 11.7 \text{ in.}$ (in.-lb units)

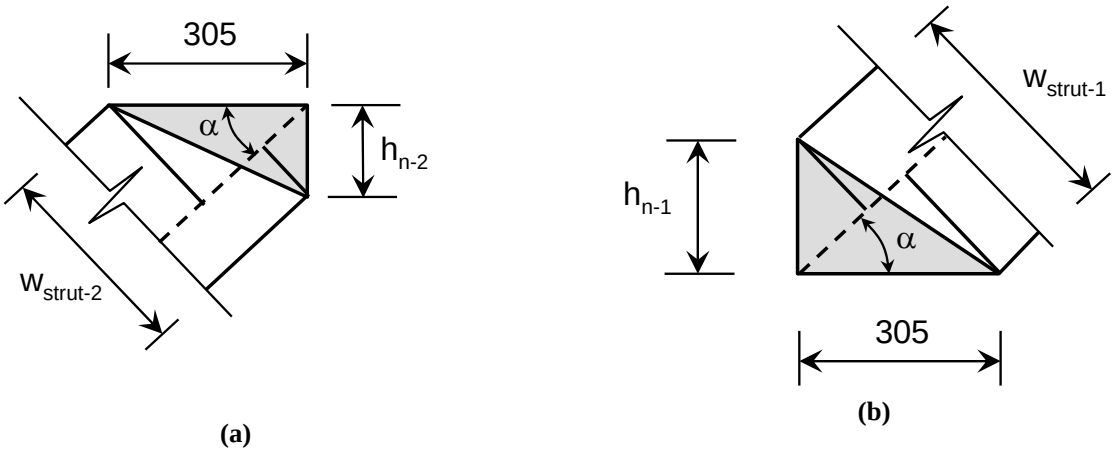


Fig. 1: Geometry of nodes and strut widths in design example (a) Node 2; and (b) Node 1 (Dimensios in millimeters; 1 in. = 25.4 mm)

Minimum web reinforcement meeting the requirements in Section A3.3 will be provided, so the strength of Strut 1-2 is calculated per Section A3.2 as:

$f_{ce(strut1-2)} = 0.85\beta_s f_c' = 0.85(0.75)(28 \text{ MPa}) = 17.9 \text{ MPa}$ (SI units)

$f_{ce(strut1-2)} = 0.85\beta_s f_c' = 0.85(0.75)(4 \text{ ksi}) = 2.55 \text{ ksi}$ (in.-lb units)

Calculate the stress on Strut 1-2 at Node 2 $f_{strut\text{-}2}$:

$f_{strut-2} = \frac{1252 \text{ kN}}{(305 \text{ mm})(296 \text{ mm})} = 13.9 \text{ MPa}$ (SI units)

$f_{strut-2} = \frac{280 \text{ kip}}{(12 \text{ in.})(11.7 \text{ in.})} = 2.0 \text{ ksi}$ (in.-lb units)

$f_{strut-2} \le f_{ce(strut1-2)} \therefore \text{OK}$

Note that there is no need to check $f_{strut\text{-}2}$ against the strength of Node 2 because $\beta_n > \beta_s$.

Determine the geometry of Node 1, as shown in Fig. 5(b).

$h_{n-1} = 2(h - d) = 2(915 \text{ mm} - 810 \text{ mm}) = 210 \text{ mm}$ (SI units)

$h_{n-1} = 2(h - d) = 2(36 \text{ in.} - 31.9 \text{ in.}) = 8.2 \text{ in.}$ (in.-lb units)

$w_{strut-1} = h_{n-1} \cos\alpha + w_{bearing} \sin\alpha$

$= (210 \text{ mm}) \cos 38.6° + (305 \text{ mm}) \sin 38.6° = 354 \text{ mm}$ (SI units)

$= (8.2 \text{ in.}) \cos 38.6° + (12 \text{ in.}) \sin 38.6° = 13.9 \text{ in.}$ (in.-lb units)

Check stresses on Node 1. With one tie being anchored, $\beta_n = 0.80$, and the allowable stress at the node is given by:

$f_{ce(1)} = 0.85\beta_n f_c' = 0.85(0.80)(28 \text{ MPa}) = 19.0 \text{ MPa}$ (SI units)

$f_{ce(1)} = 0.85\beta_n f_c' = 0.85(0.80)(4 \text{ ksi}) = 2.7 \text{ ksi}$ (in.-lb units)

and the bearing stress at the node is given by:

$f_{bearing-1} = \frac{780 \text{ kN}}{(305 \text{ mm})(305 \text{ mm})} = 8.4 \text{ MPa}$ (SI units)

$$f_{bearing-1} = \frac{175 \text{ kip}}{(12 \text{ in.})(12 \text{ in.})} = 1.2 \text{ ksi} \quad \text{(in.-lb units)}$$

$f_{bearing-1} \leq f_{ce(1)} \therefore$ OK

Similarly, the stress on the vertical face of Node 1 is given by:

$$f_{vertical-1} = \frac{F_{Tie1-4}}{bh_{n-1}} = \frac{979 \text{ kN}}{(305 \text{ mm})(210 \text{ mm})} = 15.3 \text{ MPa} \quad \text{(SI units)}$$

$$f_{vertical-1} = \frac{F_{Tie1-4}}{bh_{n-1}} = \frac{220 \text{ kip}}{(12 \text{ in.})(8.2 \text{ in.})} = 2.2 \text{ ksi} \quad \text{(in.-lb units)}$$

$f_{vertical-1} \leq f_{ce(1)} \therefore$ OK

Stress on the inclined face of Node 1 is checked against the maximum allowable stress in Strut 1-2 at Node 1, which is lower than the allowable nodal stress at Node 1.

$$f_{strut-1} = \frac{F_{strut1-2}}{bw_{strut-1}} = \frac{1252 \text{ kN}}{(305 \text{ mm})(354 \text{ mm})} = 11.6 \text{ MPa} \quad \text{(SI units)}$$

$$f_{strut-1} = \frac{F_{strut1-2}}{bw_{strut-1}} = \frac{280 \text{ kip}}{(12 \text{ in.})(13.9 \text{ in.})} = 1.7 \text{ ksi} \quad \text{(in.-lb units)}$$

$f_{strut-1} \leq f_{ce(strut1-2)} \therefore$ OK

The next step is to calculate the required area of bottom tie reinforcement $A_{s(tie\ 1\text{-}4)}$

$$A_{s(tie1-4)} = \frac{F_{tie1-4}}{f_y} = \frac{979 \text{ kN}}{420 \text{ MPa}} = 2330 \text{ mm}^2 \quad \text{(SI units)}$$

$$A_{s(tie1-4)} = \frac{F_{tie1-4}}{f_y} = \frac{220 \text{ kip}}{60 \text{ ksi}} = 3.67 \text{ in.}^2 \quad \text{(in.-lb units)}$$

Select 6 No. 22 (No. 7) bars (A_s = 2322 mm^2 (A_s = 3.6 in.^{2}) ≈ $A_{s(tie1\text{-}4)}$). Check the anchorage length for these bars. Appendix A of ACI 318-08 allows the use of an extended nodal zone for checking reinforcement anchorage, as shown in Fig. 3(a). Assuming a minimum back cover of 50 mm (2 in.), the available anchorage length is 530 mm (20.9 in.). The development length l_{dh} for a No. 22 (No. 7) hooked bar is,

$$l_{dh} = \frac{0.24 f_y}{\sqrt{f_c'}} d_b = \frac{0.24(420 \text{ MPa})}{\sqrt{28 \text{ MPa}}} (22 \text{ mm}) = 420 \text{ mm} \quad \text{(SI units)}$$

$$l_{dh} = \frac{0.02 f_y}{\sqrt{f_c'}} d_b = \frac{0.02(60{,}000 \text{ psi})}{\sqrt{4000 \text{ psi}}} (0.875 \text{ in.}) = 17 \text{ in.} \quad \text{(in.-lb units)}$$

Finally, determine the minimum web reinforcement to satisfy Section A.3.3.1 of ACI 318-08. Try 2-legged No. 10 (No. 3) stirrups @ 200 mm (8 in.) spacing and two layers of two No. 10 (No. 3) horizontal bars @ 235 mm (9.25 in.) spacing as shown in Fig. 4(b).

$$\sum \frac{A_{si}}{bs_i} \sin \gamma_i = \frac{142 \text{ mm}^2}{(305 \text{ mm})(200 \text{ mm})} \sin(90° - 38.6°) + \frac{142 \text{ mm}^2}{(305 \text{ mm})(235 \text{ mm})} \sin 38.6° = 0.0031 \quad \text{(SI units)}$$

$$\sum \frac{A_{si}}{bs_i} \sin \gamma_i = \frac{0.22 \text{ in.}^2}{(12 \text{ in.})(8 \text{ in.})} \sin(90° - 38.6°) + \frac{0.22 \text{ in.}^2}{(12 \text{ in.})(9.25 \text{ in.})} \sin 38.6° = 0.0030 \quad \text{(in.-lb units)}$$

$\sum \frac{A_{si}}{bs_i} \sin \gamma_i \geq 0.003 \therefore$ OK

Although not required, add two No. 22 (No. 7) longitudinal bars as compression reinforcement for ductility. The final beam design is shown in Fig. 4(b).